Numerical Dynamics Simulation and Steady-State Analysis of Multimode Cat States

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December 19, 2024

Abstract

We investigate the preparation and stabilization of multimode cat states in a one-dimensional array of Kerr resonators subject to parametric two-photon drive and nonlocal dissipation. By transforming the system into the plane-wave basis, we analyze the dynamics and identify a decoherence-free subspace (DFS) where dark states reside. Numerical simulations using the QuTiP confirm that the system evolves into the cat state of a collective bosonic basis, and we provide the Python code used for these simulations.

1 Introduction

Quantum superposition states, such as Schrödinger cat states, play a crucial role in quantum error correction and quantum computation. In this report, we explore a scheme to prepare and stabilize multimode cat states in a dissipative quantum system composed of a one-dimensional array of Kerr resonators. We analyze the system's dynamics under the influence of nonlocal dissipation and a two-photon drive and identify conditions under which the system reaches a steady state within a decoherence-free subspace (DFS).

2 Model

We consider a one-dimensional array of N Kerr resonators, each subjected to a parametric two-photon drive. In a frame rotating at the resonator frequency, the Hamiltonian of the system is given by

$$\hat{H} = \hat{H}_U + \hat{H}_G, \tag{1}$$

where

$$\hat{H}_U = U \sum_{j=1}^{N} \hat{a}_j^{\dagger 2} \hat{a}_j^2 \tag{2}$$

is the Kerr nonlinearity term with strength U, and

$$\hat{H}_G = G \sum_{j=1}^{N} \left(e^{-i\theta j} \hat{a}_j^{\dagger 2} + e^{i\theta j} \hat{a}_j^2 \right) \tag{3}$$

is the two-photon drive term with amplitude G and phase offset per site θ.

2.1 Transformation to Plane-Wave Basis

To simplify the analysis, we assume periodic boundary conditions $\hat{a}_{N+1} = \hat{a}_1$ and transform the Hamiltonian into the plane-wave basis using the Fourier transform:

$$\hat{b}_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{ikj} \hat{a}_j, \tag{4}$$

where $k = \frac{2\pi m}{N}$ with $m \in \{0, 1, \ldots, N-1\}$. The inverse transform is

$$\hat{a}_j = \frac{1}{\sqrt{N}} \sum_{k} e^{-ikj} \hat{b}_k. \tag{5}$$

In the plane-wave basis, the Kerr nonlinearity term becomes

$$\hat{H}_U = \frac{U}{N} \sum_{k_1,k_2,k_3,k_4} \delta_{k_1+k_2,k_3+k_4} \hat{b}_{k_1}^{\dagger} \hat{b}_{k_2}^{\dagger} \hat{b}_{k_3} \hat{b}_{k_4}, \tag{6}$$

where the Kronecker delta ensures conservation of total quasimomentum. The two-photon drive term transforms to

$$\hat{H}_G = G \sum_{k} \left(\hat{b}_k^{\dagger} \hat{b}_{\theta-k}^{\dagger} + \hat{b}_k \hat{b}_{\theta-k} \right). \tag{7}$$

2.2 Nonlocal Dissipation

The system is subject to nonlocal dissipation described by the Lindblad master equation:

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] + \gamma \sum_{j=1}^{N} \mathcal{D}[\hat{a}_j - e^{i\phi}\hat{a}_{j+1}]\hat{\rho}, \tag{8}$$

where γ is the dissipation rate, ϕ is a phase offset, and $\mathcal{D}[\hat{O}]\hat{\rho} = \hat{O}\hat{\rho}\hat{O}^\dagger - \frac{1}{2}\{\hat{O}^\dagger\hat{O}, \hat{\rho}\}$.

This nonlocal dissipation operator can be realized by coupling the adjacent two cavities to a waveguide [2].

Transforming the dissipation term into the plane-wave basis, we find

$$\gamma \sum_{j=1}^{N} \mathcal{D}[\hat{a}_j - e^{i\phi}\hat{a}_{j+1}]\hat{\rho} = \sum_{k} \gamma_k \mathcal{D}[\hat{b}_k]\hat{\rho}, \tag{9}$$

where the mode-dependent dissipation rates are

$$\gamma_k = 2\gamma \left[1 - \cos(k - \phi)\right]. \tag{10}$$

Notably, for $k = \phi$, the dissipation rate γ_k vanishes, indicating the presence of a decoherence-free subspace.

3 Solution of steady state

3.1 Steady State in Plane-wave Basis

We aim to characterize the steady states of the master equation, focusing on identifying dark states within the DFS. These dark states satisfy [3]:

$$\hat{b}_k|\Psi\rangle = 0 \quad \text{for all } k \neq \phi, \tag{11}$$

and

$$\hat{H}|\Psi\rangle = \epsilon|\Psi\rangle. \tag{12}$$

The dark state condition can be rewritten as:

$$\sum_{k} \hat{b}_k^\dagger \hat{b}_{2\phi-k}^\dagger (\hat{b}_\phi^2 - \zeta^2)|\Psi\rangle = \zeta^{*2}\left(\hat{b}_\phi^2 - \frac{\epsilon}{G}\right)|\Psi\rangle, \tag{13}$$

where $\zeta = i\sqrt{NG/U}$. For $\epsilon = G\zeta^2$, this leads to:

$$(\hat{b}_\phi - \zeta)(\hat{b}_\phi + \zeta)|\psi\rangle_\phi = 0. \tag{14}$$

We find that the eigenstates are the even and odd cat states in the mode $k = \phi$:

$$|\Psi_\phi^\pm\rangle = \mathcal{N}_\pm \left(|\zeta\rangle_\phi \pm |-\zeta\rangle_\phi\right), \tag{15}$$

where $|\zeta\rangle_\phi$ is a coherent state of mode ϕ, and $\mathcal{N}_\pm$ are normalization constants.

3.2 Multimode Cat States in Local Basis

Transforming back to the local basis, the dark states (multimode cat states) are

$$|C^\pm\rangle = \mathcal{N}_\pm \left(\bigotimes_{j=1}^{N} |\zeta_j\rangle_j \pm \bigotimes_{j=1}^{N} |-\zeta_j\rangle_j \right), \tag{16}$$

where

$$\zeta_j = \left(\frac{\zeta}{\sqrt{N}}\right) e^{-i\phi j}. \tag{17}$$

4 Steady-State Preparation of Multimode Cat States

To prepare the multimode cat state $|C^+\rangle$, we initialize the system in the vacuum state $\hat{\rho}_{\text{in}} = |000\rangle\langle 000|$ and let it evolve under the master equation. We perform numerical simulations using the QuTiP library [1] to solve the master equation.

The simulations demonstrate that the populations of modes with $k \neq \phi$ remain zero, leading the system to evolve into a steady-state cat state, as illustrated in Figs. 1 and 2.

By transforming the system to the local basis and performing a partial trace, we obtain the Wigner function for the local mode, depicted in Figs. 3, 4, and 5. The results show that the coherent states of different modes are phase-shifted by ϕ, aligning with theoretical predictions (Eq. 17). Additionally, the Wigner function of a single local mode does not exhibit the interference fringes typical of a cat state. This absence is due to the system being in a multimode entangled cat state(Eq. 16), analogous to a GHZ state in the coherent basis. After performing the partial trace, the reduced state of a single mode is a mixed state composed of two coherent states.

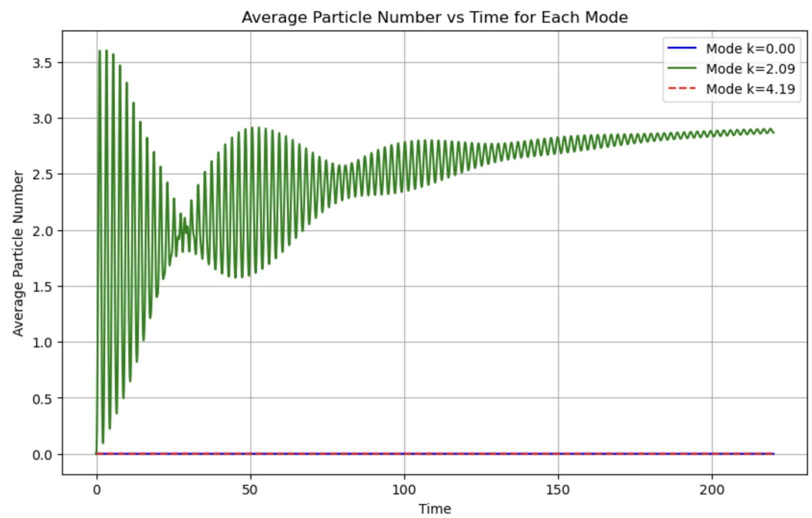

Figure 1: Average photon number evolution of three mode

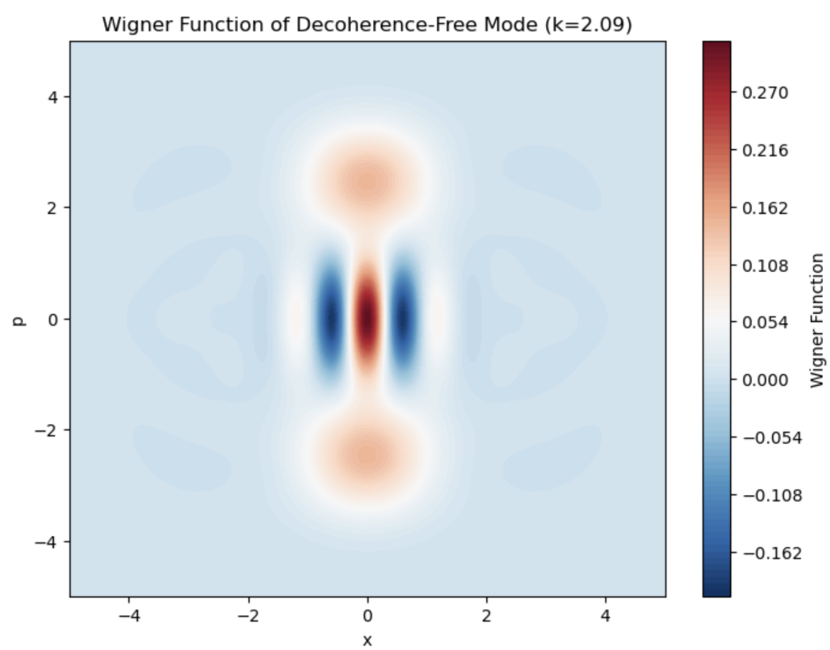

Figure 2: Wigner function of the DFI mode

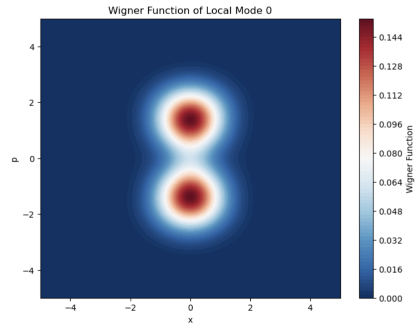

Figure 3: Wigner function of the local mode 0

Figure 4: Wigner function of the local mode 1

Figure 5: Wigner function of the local mode 2

5 Conclusion

We have demonstrated that multimode cat states can be prepared and stabilized in a one-dimensional array of Kerr resonators with nonlocal dissipation and two-photon drive. The identification of a decoherence-free subspace and the analysis of the system's dynamics provide a solid foundation for the experimental realization of such states.

References

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[3] Petr Zapletal, Andreas Nunnenkamp, and Matteo Brunelli. Stabilization of multimode schrödinger cat states via normal-mode dissipation engineering. *PRX Quantum*, 3(1):010301, 2022.